

AdS-MAXWELL BF THEORY AS A MODEL OF GRAVITY AND BI-GRAVITY

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This article presents an extended model of gravity obtained by gauging the AdS-Maxwell algebra. It involves additional fields that shift the spin connection, leading effectively to theory of two independent connections. Extension of algebraic structure by another tetrad gives rise to the model described by a pair of Einstein equations.

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It was shown in the recent works^{1,2} that it is possible to extend the Poincare algebra by extra charges, which are related to a constant background Maxwell field.³ Such algebra is called Maxwell algebra. Unfortunately Maxwell algebra can not be used to gauging the theory in geometrical way,⁴ because, like the original Poincare algebra, it does not possess a non-degenerate ad-invariant inner product. Therefore as a first step to gauge Maxwell algebra one has to form its (Anti) de Sitter extension, which reads⁵⁻⁷

$$\begin{aligned}
 [\mathcal{M}_{ab}, \mathcal{M}_{cd}] &= -i(\eta_{ac}\mathcal{M}_{bd} + \eta_{bd}\mathcal{M}_{ac} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad}), \\
 [\mathcal{M}_{ab}, \mathcal{P}_c] &= -i(\eta_{ac}\mathcal{P}_b - \eta_{bc}\mathcal{P}_a), \\
 [\mathcal{P}_a, \mathcal{P}_b] &= i(\mathcal{M}_{ab} + k\mathcal{Z}_{ab}), \quad [\mathcal{Z}_{ab}, \mathcal{P}_c] = 0 \\
 [\mathcal{M}_{ab}, \mathcal{Z}_{cd}] &= -i(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}), \\
 [\mathcal{Z}_{ab}, \mathcal{Z}_{cd}] &= +ik(\eta_{ac}\mathcal{Z}_{bd} + \eta_{bd}\mathcal{Z}_{ac} - \eta_{ad}\mathcal{Z}_{bc} - \eta_{bc}\mathcal{Z}_{ad}).
 \end{aligned} \tag{1}$$

with $k = +1$ for dS-Maxwell, $k = -1$ for AdS-Maxwell algebra and $a, b, \dots = 0, \dots, 3$.

Gauging this algebra one gets the connection

$$\mathbb{A}_\mu = \frac{1}{2}\omega_\mu^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}e_\mu^a\mathcal{P}_a + \frac{1}{2}h_\mu^{ab}\mathcal{Z}_{ab}, \tag{2}$$

and its curvature

$$\mathbb{F}_{\mu\nu} = \frac{1}{2}F_{\mu\nu}^{ab}\mathcal{M}_{ab} + \frac{1}{\ell}T_{\mu\nu}^a\mathcal{P}_a + \frac{1}{2}G_{\mu\nu}^{ab}\mathcal{Z}_{ab} \tag{3}$$

which makes it possible to construct a gauge invariant action in the form of a

constrained topological BF theory.⁸ This action reads⁷

$$\begin{aligned}
 16\pi S(A, B) = & \int 2(B^{a4} \wedge F_{a4} - \frac{\beta}{2} B^{a4} \wedge B_{a4}) \\
 & + B^{ab} \wedge F_{ab} - \frac{\beta}{2} B^{ab} \wedge B_{ab} - \frac{\alpha}{4} \epsilon^{abcd} B_{ab} \wedge B_{cd} \\
 & + C^{ab} \wedge G_{ab} - \frac{\rho}{2} C^{ab} \wedge C_{ab} - \frac{\sigma}{4} \epsilon^{abcd} C_{ab} \wedge C_{cd} \\
 & - \beta C^{ab} \wedge B_{ab} - \frac{\alpha}{2} \epsilon^{abcd} C_{ab} \wedge B_{cd}, \tag{4}
 \end{aligned}$$

where C^{ab}, B^{ab}, B^{a4} are auxiliary fields. By construction the action (4) is manifestly diffeomorphism-invariant and possess local Lorentz and Maxwell symmetries, but the translational part of (A)dS-Maxwell symmetry, generated by P is broken.^{4,8} After eliminating the auxiliary fields by solving their field equations the action (4) takes form

$$\begin{aligned}
 16\pi S(\omega, h, e) = & \int \left(\frac{1}{4} M^{abcd} F_{ab} \wedge F_{cd} - \frac{1}{\beta \ell^2} T^a \wedge T_a \right) \\
 & + \int \frac{1}{4} N^{abcd} (G_{ab} + F_{ab}) \wedge (G_{ab} + F_{ab}), \tag{5}
 \end{aligned}$$

where

$$M^{ab}{}_{cd} = \frac{\alpha}{\alpha^2 + \beta^2} \left(\frac{\beta}{\alpha} \delta_{cd}^{ab} - \epsilon^{ab}{}_{cd} \right)$$

and

$$N^{abcd} = \frac{(\sigma - \alpha)}{(\sigma - \alpha)^2 + (\rho - \beta)^2} \left(\frac{\rho - \beta}{\sigma - \alpha} \delta^{abcd} - \epsilon^{abcd} \right)$$

Since the last term in (5) is a topological invariant^a this action describe pure gravity.

The algebra (1) can be further extended by adding yet another translational generator \mathcal{R}_a with the commutation relations

$$[\mathcal{R}_a, \mathcal{R}_b] = i\mathcal{Z}_{ab}, \quad [\mathcal{P}_a, \mathcal{R}_c] = 0, \tag{6}$$

$$[\mathcal{M}_{ab}, \mathcal{R}_c] = -i(\eta_{ac}\mathcal{R}_b - \eta_{bc}\mathcal{R}_a), \quad [\mathcal{Z}_{ab}, \mathcal{R}_c] = -i(\eta_{ac}\mathcal{R}_b - \eta_{bc}\mathcal{R}_a). \tag{7}$$

The gauge connection becomes

$$\mathbb{A}_\mu = \frac{1}{2} \omega_\mu{}^{ab} \mathcal{M}_{ab} + \frac{1}{\ell} e_\mu^a \mathcal{P}_a + \frac{1}{2} h_\mu^{ab} \mathcal{Z}_{ab} + \frac{1}{\ell'} f_\mu^a \mathcal{R}_a, \tag{8}$$

and the gauge curvatures take the form

$$F_{\mu\nu}^{ab} = R_{\mu\nu}^{ab} + \frac{1}{\ell^2} (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b), \quad T_{\mu\nu}^a = D_\mu^\omega e_\nu^a - D_\nu^\omega e_\mu^a, \tag{9}$$

$$\begin{aligned}
 G_{\mu\nu}^{ab} = & D_\mu^\omega h_\nu^{ab} - D_\nu^\omega h_\mu^{ab} - \frac{1}{\ell^2} (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b) \\
 & + (h_\mu^{ac} h_{\nu c}{}^b - h_\nu^{ac} h_{\mu c}{}^b) - \frac{1}{\ell^2} (f_\mu^a f_\nu^b - f_\nu^a f_\mu^b), \tag{10}
 \end{aligned}$$

$$Y_{\mu\nu}^a = D_\mu^\omega f_\nu^a - D_\nu^\omega f_\mu^a + h_\mu^{ab} f_{\nu b} - h_\nu^{ab} f_{\mu b}. \tag{11}$$

^aIt is combination of the Euler and Pontryagin invariant for shifted connection $\omega + h$.

The total action is slightly different from (4) and equals

$$S = S^{(AdS-Maxwell)} + 2 C^a \wedge Y_a. \quad (12)$$

The crucial observation is that after adding the new generator, the action (5) is not topological anymore because the expression (10) has changed. If the connection is shifted by the Maxwell field i.e., $\varpi = \omega + h$, which is equivalent to changing the basis of the Lie algebra by $\mathcal{M} \rightarrow \mathcal{M} - \mathcal{Z}$, the last term of (5) becomes another Einstein action for connection ϖ , tetrad f , and with cosmological constant Λ'

$$S_{E_2} = \frac{1}{64\pi G} \int \epsilon_{abcd} \left(H_{\mu\nu}[\varpi]^{ab} f_\rho^c f_\sigma^d - \frac{\Lambda'}{3} f_\mu^a f_\nu^b f_\rho^c f_\sigma^d \right) \epsilon^{\mu\nu\rho\sigma}, \quad (13)$$

where $H_{\mu\nu}[\varpi]^{ab}$ is curvature of the shifted connection, and $\Lambda' = 3/\ell'$ (see eq. 8). It follows from the field equation derived from (12) that the torsion of the connection ϖ vanishes

$$df^a + \varpi^a_b \wedge f^b = \tilde{T}^a = 0. \quad (14)$$

It is worth recalling that the model of Einstein pair was previously discussed in a similar context in Ref. 9, but there the existence of another tetrad, and the torsion equation was introduced by hand. Here the tetrad emerges naturally as a result of the algebra enlargement, and satisfies field equations, which forces torsion to vanish. However, there is not interaction term between tetrads, which is crucial in bi-gravity theories such as the $f - g$ model.¹⁰ It was pointed out in Ref. 9 that one can add such gauge invariant term, but it is not possible to obtain it by enlarging the (A)dS algebras. We will present a more detailed discussion of these algebras and their physical applications in a separate paper.

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